Cographic Element Splitting Matroids of regular Matroids

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Abstract

In this paper, we give a forbidden minor characterization for element splitting matroids of a regular matroid to be cographic. The main theorem states that the element splitting operation, by each pair of elements, on a regular matroid yields a cographic matroid if and only if it has no $M(K_4)$-minor and has no $M(A_1)$-minor for a specific 6×11 binary matrix A_1.

AMS Subject Classifications: 05B35; 05C50; 05C83

Keywords: *Binary matroid, regular matroid, cographic matroid, minor, splitting operation, element splitting operation*

1. Introduction

The element splitting operation for a binary matroid is defined [2] as follows: Let A be a matrix over $GF(2)$ that represents the matroid M. Suppose that x and y are distinct elements of M. Let $A'_{x,y}$ be the matrix that is obtained by adjoining an extra row to A with this row being zero everywhere except in the columns corresponding to x and y where it takes the value 1 and then adjoining an extra column (corresponding to the new element) with this column being zero everywhere except in the last row where it takes the value 1. Suppose $M'_{x,y}$ is the matroid represented by the matrix $A'_{x,y}$. The transition from M to $M'_{x,y}$ is called an element splitting operation. The matroid $M'_{x,y}$ is called the *element splitting matroid.*

If M is the cycle matroid of a graph G of Figure 1 then $M'_{x,y}$ is the cycle matroid of the graph $G'_{x,y}$ of Figure 1.

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ALMOST SELF-COMPLEMENTARY 3-UNIFORM HYPERGRAPHS

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Abstract

It is known that self-complementary 3-uniform hypergraphs on n vertices exist if and only if n is congruent to 0, 1 or 2 modulo 4. In this paper we define an almost self-complementary 3-uniform hypergraph on n vertices and prove that it exists if and only if n is congruent to 3 modulo 4. The structure of corresponding complementing permutation is also analyzed. Further, we prove that there does not exist a regular almost self-complementary 3-uniform hypergraph on n vertices where n is congruent to 3 modulo 4, and it is proved that there exist a quasi regular almost self-complementary 3-uniform hypergraph on n vertices where n is congruent to 3 modulo 4.

Keywords: uniform hypergraph, self-complementary hypergraph, almost complete 3-uniform hypergraph, almost self-complementary hypergraph, quasi regular hypergraph.

2010 Mathematics Subject Classification: 05C65.

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THE EXISTENCE OF QUASI REGULAR AND BI-REGULAR SELF-COMPLEMENTARY 3-UNIFORM HYPERGRAPHS

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Abstract

A k-uniform hypergraph $H = (V; E)$ is called self-complementary if there is a permutation $\sigma : V \to V$, called a complementing permutation, such that for every k-subset e of V, $e \in E$ if and only if $\sigma(e) \notin E$. In other words, H is isomorphic with $H' = (V; V^{(k)} - E)$. In this paper we define a bi-regular hypergraph and prove that there exists a bi-regular self-complementary 3-uniform hypergraph on n vertices if and only if n is congruent to 0 or 2 modulo 4. We also prove that there exists a quasi regular self-complementary 3-uniform hypergraph on n vertices if and only if n is congruent to 0 modulo 4.

Keywords: self-complementary hypergraph, uniform hypergraph, regular hypergraph, quasi regular hypergraph, bi-regular hypergraph.

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MAGIC OF SEMIGRAPHS

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Abstract. In this paper, the concept of semigraph is illustrated with lot of examples. Semigraphic matrices and matrix semigraphs are defined. A vector space approach to obtain semigraphs is established. Magic and antimagic semigraphs are studied.

Keywords: Semigraphs and Examples.

Mathematical Subject Classification: 05C15, 05C99.

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A proof of Frankl's union-closed sets conjecture for dismantlable lattices

Vinayak Joshi, B. N. Waphare, and S. P. Kavishwar

Abstract. In this paper, we prove Frankl's Union-Closed Sets Conjecture for the class of dismantlable lattices, a more general class than the class of planar lattices. As a consequence of this result, we also prove that an upper semimodular lattice with breadth at most two satisfies the Conjecture.

1. Introduction

In 1979, Peter Frankl conjectured the following, known as the Union-Closed Sets Conjecture or Frankl's Conjecture.

Union-Closed Sets Conjecture 1.1. *Let $\mathcal{F}$ be a collection of subsets of a finite set X such that $F \cup G \in \mathcal{F}$ holds for all $F, G \in \mathcal{F}$, that is, $\mathcal{F}$ is a union-closed family. If $|\mathcal{F}| \geq 2$, then there is an element x in X such that at least $|\mathcal{F}|/2$ members $F \in \mathcal{F}$ satisfy $x \in F$.*

Poonen [9] seems to be the first one who formulated the Conjecture in the language of lattice theory; see also Stanley [13, p. 161].

Frankl's Conjecture 1.2 (Poonen [9], Stanley [13])**.** *In every finite lattice L with $|L| \geq 2$, there is a join-irreducible element j (that is $j = a \vee b \Rightarrow j = a$ or $j = b$) such that $|\{x \in L \colon j \leq x\}| \leq |L|/2$.*

This conjecture is equivalent to the Union-Closed Sets Conjecture. In [9], Poonen proved that this conjecture is true for finite geometric lattices and relatively complemented lattices.

Despite its elementary statement, the Union-Closed Sets Conjecture is considered to be one of the most difficult problems in extremal set theory. The conjecture remains unsolved, though some partial results have been obtained. Mainly, two approaches were used to solve the conjecture, one using pure combinatorial arguments and the second by the use of lattice theoretic methods. In recent years, there has been a greater use of lattice theoretic methods to solve the conjecture for special classes of lattices; see Abe and Nakano [2], Czédli and Schmidt [5], Shewale, Joshi and Kharat [12].

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ORIGINAL PAPER

New Numerical Methods for Solving Differential Equations

Yogita Sukale[1,2] · Varsha Daftardar-Gejji[1]

Abstract New numerical methods have been developed for solving ordinary differential equations (with and without delay terms). In this approach existing methods such as trapezoidal rule, Adams Moulton methods are improvised using new iterative method (Daftardar-Gejji and Jafari in J Math Anal Appl 316(2):753–763, 2006). Further consistency of the new methods is checked along with error and stability analysis. Numerical examples are presented to illustrate these methods and compared with the existing methods. It is observed that the newly proposed methods yield more accurate results than the existing methods.

Keywords New iterative method (NIM) · Trapezoidal rule · Adams Moulton method · Adams Bashforth method · Delay differential equations

Introduction

Ordinary differential equations (ODEs) frequently occur as mathematical models in many branches of science, engineering and economy. Many models have no closed form solutions and hence it is a need to seek approximate solutions by means of numerical methods. Delay differential equations (DDEs) form an important class of dynamical systems. They often arise in natural systems where delayed effects are important e.g. population dynamics, immunology, physiology, communication networks, control systems etc. [1,5–9,11]. DDEs exhibit rich dynamics due to the fact that they operate on infinite dimensional spaces. Consequently analytical calculations in case of DDEs are more difficult than ODEs and generally one has to resort to numerical methods for solving them.

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On Hopf bifurcation in fractional dynamical systems

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ABSTRACT

Fractional order dynamical systems admit chaotic solutions and the chaos disappears when the fractional order is reduced below a threshold value [1]. Thus the order of the dynamical system acts as a chaos controlling parameter. Hence it is important to study the fractional order dynamical systems and chaos. Study of fractional order dynamical systems is still in its infancy and many aspects are yet to be explored.

In pursuance to this in the present paper we prove the existence of fractional Hopf bifurcation in case of fractional version of a chaotic system introduced by Bhalekar and Daftardar-Gejji [2]. We numerically explore the (A, B, α) parameter space and identify the regions in which the system is chaotic. Further we find (global) threshold value of fractional order α below which the chaos in the system disappears regardless of values of system parameters A and B.

1. Introduction

Fractional order dynamical systems are gaining popularity due to their widespread applications [3]. The study of chaotic dynamical systems of fractional order was initiated by Grigorenko and Grigorenko [1] wherein fractional ordered Lorenz system was studied. It was shown that the order of the derivative acts as a chaos controlling parameter and below a threshold value of α, chaos disappears. These simulations were done by keeping rest of the system parameters fixed. Since then there has been increasing interest in this topic and a large number of contributions have appeared in the literature which deal with fractional versions of various chaotic systems including Chen System [4], Rössler system [5], Liu system [6], financial system [7] and so on [3].

In spite of the extensive numerical work, our understanding of fractional systems is not complete and very few analytical results have been obtained. The first important result obtained regarding stability analysis of fractional systems is due to Matignon [8]. Some of the important results regarding stability of fractional systems have been summarized by Li and Zhang [9].

The system introduced by Bhalekar and Daftardar-Gejji (BG system) has been shown to be chaotic for certain values of parameters [2]. Further the forming mechanism of this system is discussed by Bhalekar [10]. The synchronization and anti-synchronization of Bhalekar – Gejji system and Liu system is done by Singh et al. [11]. The Hopf bifurcation in integer order Bhalekar – Gejji system has been explored by Aqeel and Ahmad [12]. In the present paper we prove existence of Hopf bifurcation in fractional version of BG system and explore the parameter space numerically.

The paper is organized as follows. Section 2 comprises of preliminaries and notations. Section 3 deals with fractional Hopf bifurcation and proves its existence for fractional BG system. Sections 4, 5 and Section 6 contain numerical explorations for various system parameters. Conclusions are summarized in Section 7.

2. Preliminaries

In this section, we introduce notations, definitions and preliminaries pertaining to fractional calculus and stability of fractional dynamical systems [1,9,13,14]. $N_r(a)$ denotes the neighborhood of point $a \in \mathbb{R}^n$ having radius $r > 0$. $\|.\|$ denotes standard Euclidean norm on $\mathbb{R}^n$.

Definition 1 [15]. The fractional integral of order $\alpha > 0$ of a real valued function f is defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau.$$

Definition 2 [15]. Caputo fractional derivative of order $\alpha > 0$ of a real valued function f is defined as

$$D^\alpha f(t) = I^{m-\alpha} D^m f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f^{(m)}(\tau) d\tau,$$
$$m-1 < \alpha < m,$$
$$= f^{(m)}(t), \quad \alpha = m, \; m \in \mathbb{N}.$$

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